Saturation and hadronic cross sections at very high energies*

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We propose a simple model for the total $pp/p\bar{p}$ cross section, which is a generalization of the minijet model with the inclusion of a window in the p_T-spectrum associated to the saturation physics. Our model implies a natural cutoff for the perturbative calculations which modifies the energy behavior of this component, so that it satisfies the Froissart bound. Including the saturated component, we obtain a satisfactory description of the very high energy experimental data.

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Long ago a QCD based explanation for the growth of the hadronic cross sections was proposed by Gaisser and Halzen [1]. In their approach, called minijet model, the total cross section can be decomposed as $\sigma_{tot} = \sigma_0 + \sigma_{pQCD}$ where σ_0 characterizes the nonperturbative contribution and σ_{pQCD} is calculable in perturbative QCD. Unfortunately, this approach implies a power-like energy behavior for the total cross section, violating the Froissart bound. Several attempts were made to reduce this too fast growth [2].

At high energies the small-x gluons in a hadron wavefunction should form a Color Glass Condensate (CGC) [3]. This new state of matter is characterized by gluon saturation and by a typical momentum scale, the saturation scale Q_s, which determines the critical line separating the linear and saturation regimes of the QCD dynamics.

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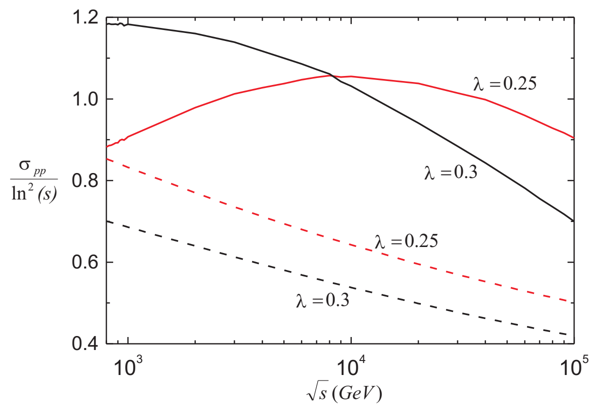

Fig. 1. Perturbative (solid lines) and saturated components (dashed lines) of the total cross section (normalized by $\ln^2 s$)

Some attempts to reconcile the QCD parton picture with the Froissart limit using saturation physics were proposed in recent years [4]. Here we generalize the minijet model assuming the existence of a saturation window between the nonperturbative and perturbative regimes of QCD, which grows when the energy increases, since Q_s grows with the energy. The cross section is then written as:

$$\sigma_{tot} = \sigma_0 + \sigma_{sat} + \sigma_{pQCD} \tag{1}$$

where the saturated component, σ_{sat}, contains the dynamics of the interactions at scales lower than the saturation scale. In our approach *the saturation scale is a cutoff at low transverse momenta of the perturbative cross section*, σ_{pQCD}, which is given by:

$$\sigma_{pQCD} = \frac{1}{2} \int_{Q_s^2} dp_T^2 \sum_{i,j} \int dx_1\, dx_2\;\; f_i(x_1, p_T^2)\; f_j(x_2, p_T^2)\; \hat{\sigma}_{ij} \tag{2}$$

where $f_i(x, Q^2)$ is the parton density of the species i, with fractional momentum x_1 (or x_2) in the proton and $\hat{\sigma}_{ij}$ is the elementary parton-parton cross section. We have used the MRST parton distributions [5]. The saturation scale is given by $Q_s^2(x) = Q_0^2\,(\frac{x_0}{x})^{\lambda}$, where the parameters $Q_0^2 = 0.3$ GeV^2 and $x_0 = 0.3 \times 10^{-4}$ were fixed by fitting the *ep* HERA data. Following [6] we take $x = \frac{q_0^2}{s}$ and $q_0 = 1.4$ GeV. Therefore we have

$$Q_s^2(s) \propto s^{\lambda}$$

.

In Fig. 1 we show in arbitrary units the energy behavior of the ratio $\sigma_{pQCD}/\ln^2 s$ (solid lines) and $\sigma_{sat}/\ln^2 s$ (dashed lines) for two choices of λ. As it can be seen the choice $\lambda = 0.25$ leads to a fast growth of σ_{pQCD} until $\sqrt{s} = 10^4$ GeV. From this point on, it grows slower than $\ln^2 s$. A slight increase in λ ($= 0.3$) is enough to tame the growth of σ_{pQCD} already at $\sqrt{s} \simeq 10^3$ GeV. On the other hand, a decrease in λ ($= 0.1$) would postpone the fall of the ratio to very high energies $\sqrt{s} \simeq 10^6$ GeV. Although the energy at which the behavior of the cross section becomes "sub-Froissart" may depend on λ, one conclusion seems very robust: *once λ is finite, at some energy the growth of the cross section will become weaker than $ln^2 s$.*

For the saturated component we shall use the model proposed in Ref. [6]:

$$\sigma_{sat} = \int d^2 r_\perp |\Psi_p(r_\perp)|^2 \sigma_{dip}(x, r_\perp) \tag{3}$$

where the proton wave function Ψ_p is chosen to be a gaussian with the typical size of the proton [7] and the dipole-proton cross section reads:

$$\sigma_{dip}(r_\perp, x) = 2 \int d^2 b \mathcal{N}(x, r_\perp, b) \ . \tag{4}$$

We take the dipole scattering amplitude from [8] (we call it IIM) and, following [6], introduce the b dependence by witting:

$$\mathcal{N}(x, r_\perp, b) \ = \ 1 \ - \ e^{- \kappa \frac{S(b)}{S(0)}} \tag{5}$$

where the parameter κ is related to the $b = 0$ solution through $\kappa = -ln[1 - \mathcal{N}(b = 0)]$. In (5), the profile function is assumed to be $S(b) = e^{(-b^2/R_p^2)}$, where $R_p = 0.7$ fm is the proton radius.

In Fig. 2 we present our results for the total cross section for different values of λ and compare them with experimental data. For references and details see [7]. σ_0 was assumed to be energy independent [9], important only at lower energies and therefore was not included in our calculations. There is only a small range of values of λ which allow us to describe the experimental data. If, for instance, $\lambda = 0.4$ the resulting cross section is very flat and clearly below the data, while if $\lambda = 0.1$ (not shown in figure) the cross section grows very rapidly deviating strongly from the experimental data. The best choice for λ is in the range $0.25 - 0.30$, which is exactly the range predicted in theoretical estimates using CGC physics and usually obtained by the saturation models for the *ep* HERA data. In [7] we have replaced the IIM dipole cross section by the more modern ones given in [10] but the results do not change very much.

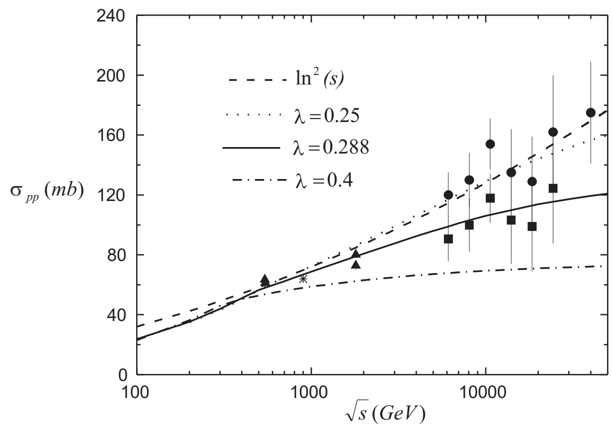

Fig. 2. Energy behavior of the total $pp/p\bar{p}$ cross section for different values of λ.

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